Relativistic self-focusing of ultraintense laser pulses in inhomogeneous underdense plasmas

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We investigate the propagation of high-intensity short laser pulses in a plasma. A two-parameter perturbation expansion is used to consistently treat the nonlinear mass variation and ponderomotive contribution to self-focusing, including plasma inhomogeneity. An analytical expression for the critical power is given and it is found that it greatly depends on the plasma inhomogeneity. A tailoring of the preformed plasma is suggested in order to obtain a strong reduction of the critical power. The temporal evolution of the pulse and plasma dynamics are also considered.

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New laser technology currently permits the achievement of focused intensities as high as 10^{18} W/cm^2 and short pulses of 100 fs to 1 ps, with associated powers in the range of 1 to 50 TW [1–3]. This development has strongly motivated the investigation of physical effects in laser-matter interaction in the relativistic electron energy range [4]. Of special interest is the study of the self-focusing of a short laser pulse in a relativistic plasma. This mechanism is very important as it would produce ultrahigh laser intensity ($10^{19}-10^{20}$ W/cm^2) over large distances compared to the usual Rayleigh length determined by the natural diffraction. As a result, relativistic self-focusing would make easier the observation of expected physical effects such as harmonic generation by relativistic electrons, huge magnetic fields generated by circularly polarized laser pulses in a plasma, and frequency upshifting of laser pulses in a plasma.

In this Brief Report, we highlight the relativistic self-focusing of an intense short laser pulse in a cold underdense plasma self-generated by the leading edge of the laser pulse in a gas. The plasma is generated by multiphoton or tunnel ionization of the gas. Thermal effects are neglected. Most of the previous theoretical calculations have considered homogeneous plasmas. This Brief Report emphasizes the importance of plasma inhomogeneity on the laser critical power in relativistic self-focusing. We study the modifications of the plasma refractive index due to two mechanisms: the relativistic variation of the electron mass and, second, the ponderomotive force, under the presence of an ultra intense laser field. The first mechanism has the effect of increasing the refractive index by decreasing the electron plasma frequency ($\omega_{p_e}^2 \equiv 4\pi q_e^2 n_e/m_e \equiv \omega_{p_0}^2/\gamma$; $m_e=\gamma m_0$, m_0 being the electron rest mass, γ the Lorentz relativistic factor, and n_e the electronic density through the relativistic mass correction). The ponderomotive force is responsible for the expulsion of the electrons from the laser channel which lowers the local electronic density n_e, and therefore enhances the former effect.

We have developed a two-parameter perturbation approach, valid in principle for arbitrary values of the electromagnetic field amplitude, which has been shown to be adequate to consistently treat the relativistic self-focusing of an intense laser pulse by an underdense ($\omega_p<\omega$; ω being the laser frequency) inhomogeneous plasma.

In the following discussion, we restrict our analysis to a pulse duration τ such that $\tau \gg \omega_{p_0}^{-1} \equiv \tau_p/2\pi$, consider underdense plasma $\delta=\omega_p/\omega<1$, and also $\tau \ll \tau_i$, i.e., the ions are considered immobile. τ_i is the ion characteristic period. This allows the introduction of two small temporal parameters δ and $\delta'=\omega_{p_0}\tau$ that are related to the different time scales ($\tau \gg \tau_p \gg T=2\pi/\omega$). Similarly, spatial scale lengths are defined by the parameters $\alpha_e=\lambda/2\pi L_e$ and $\alpha_n=\lambda/2\pi L_n$, as the ratio of the laser wavelength (λ) to the field (L_e) and electronic density (L_n) gradient lengths, respectively. A consistent expansion of the full nonlinear current in the parameters α and δ is thus made possible. The different space-time scales of the problem allow a special treatment of both the Maxwell equations and the fluid equations. The latter are the momentum and continuity equations given by

$$\frac{\partial \mathbf{p}}{\partial t}=\frac{\partial}{\partial t}(m_0\gamma\mathbf{V})=q_e\mathbf{E}+\left[-m_0\mathbf{V}\cdot\nabla(\gamma\mathbf{V})+q_e\frac{\mathbf{V}}{c}\times\mathbf{B}\right], \tag{1}$$

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{V})=0, \tag{2}$$

where $\gamma=(1+q^2)^{1/2}$, with $q^2=E^2(\mathbf{r},t)/E_c^2$, $E_c=(m_0\omega_c/q_e)$ being the Compton field. This expression for γ can be shown to be exact to order α^2q^4. Equations (1) and (2) combined with Maxwell's equations (with charge and current densities being $\rho=q_en$ and $\mathbf{j}=q_en\mathbf{V}$, respectively) lead to the $\mathbf{E}$ field propagation equation,

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}-\nabla(\nabla\cdot\mathbf{E})=-\frac{4\pi}{c}\frac{\partial \mathbf{j}}{\partial t}. \tag{3}$$

An approximate solution of the combined set of Eqs. (1), (2), and (3) may be obtained by the multiple-scale-expansion method which is a perturbative expansion in α and δ in the form $a=\sum_{j=0}^{\infty}a_j$, where a stands for the quantities n, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{p}$, $\mathbf{V}$, γ.

The fluid ponderomotive force is given in our case by

$$\mathbf{f}_p = \langle -m_0(\mathbf{V}\cdot\nabla)(\gamma\mathbf{V}) + q_e\mathbf{V}/c\times\mathbf{B}\rangle \tag{4}$$

or

$$\mathbf{f}_p = -m_0c^2\nabla(\gamma-1)\ , \tag{4'}$$

where $\mathbf{V}$ is the fluid velocity and $\langle\ \rangle$ is the time average over an optical period T.

From Eqs. (1) and (2) we have

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{q_e^2}{m_0}\left[\frac{n}{\gamma}\right]\mathbf{E} + q_e\left[\frac{\partial n}{\partial t} - \frac{n}{\gamma}\frac{\partial\gamma}{\partial t}\right]\mathbf{V} + \frac{q_e}{m_0}\left[\frac{n}{\gamma}\right]f_p \tag{5}$$

and to third order the current is

$$\mathbf{j}^{(1)} = q_e n_0\mathbf{V}_1\ ;$$
$$\mathbf{j}^{(2)} = q_e(n_1\mathbf{V}_1 + n_0\mathbf{V}_2)\ ; \tag{6}$$
$$\mathbf{j}^{(3)} = q_e(n_2\mathbf{V}_1 + n_1\mathbf{V}_2 + n_0\mathbf{V}_3)\ .$$

In Eqs. (5) and (6) the first term is the leading term up to corrections of order δ (or higher). Hence, a scalar treatment of the dielectric constant is adequate for underdense plasmas. The electronic density perturbation n_2 is evolving in the time scale of τ_p (slow),

$$\left[\frac{\partial^2}{\partial t^2} + \frac{\omega_{p_0}^2}{\gamma}\right]n_2 = \nabla\cdot\left[\frac{n_0\mathbf{f}_p}{m_0\gamma}\right] + C\ . \tag{7}$$

On the right-hand side (rhs) of Eq. (7), C is a term which is zero for the quasistatic approximation (QSA) ($\partial/\partial t = 0$ on the τ_p scale), and it is δ smaller than the first term otherwise. Therefore the QSA solution of Eq. (7) is

$$\frac{n_2(\mathbf{r})}{n_0(0)} = -\alpha_p^2\left[\frac{n_0(\mathbf{r})}{n_0(0)}\left[\nabla^2\gamma - \frac{(\nabla\gamma)^2}{\gamma}\right] + \frac{\nabla\gamma\cdot\nabla n_0}{n_0(0)}\right]\ , \tag{8}$$

where $\alpha_p = \lambda_p/L_e$ and $\lambda_p = c/\omega_{p_0}$ is the classical plasma skin depth.

In Eq. (8) the first term on the rhs is the same as described in Refs. [5, 6, 7]; it is related to $\nabla\cdot\mathbf{f}_p$. The third term is the coupling of the ponderomotive force with the gradient of the electronic density. Calculations show that the first term is dominant over the second for $E \lesssim 2E_c$ (Gaussian pulses). The contribution of each term to the nonlinear current may be estimated from the following values: $|V_1| \simeq E/\gamma E_c\omega$; $\max|V_2/V_1| \simeq \alpha_e E/\delta\gamma E_c$; $|V_2| = |q_eE_2 + f_p|/m_0\gamma\omega_p$ (in the static approximation, $V_2 = 0$ since the induced self-consistent electrostatic force q_eE_2 balances the ponderomotive force $\mathbf{f}_p$); $|n_1/n_0| \simeq \alpha_n(E/E_c)$; $|n_2/n_0| \ll (\alpha_p)^2$. The nonlinear current given by Eq. (6) is $\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(3)}$ with $\mathbf{j}^{(3)} = q_e n_2\mathbf{V}_1$. This is the equivalent of the nonperturbative Eq. (5) with $n = n_0 + n_2$.

An approximation of Eq. (3) is given by the classical longitudinal approximation for the envelope equation [8]

$$2ik\left[\frac{\partial}{\partial z} + \frac{1}{V_g}\frac{\partial}{\partial t}\right]\mathbf{E} = \nabla_\perp^2\mathbf{E} + \frac{\omega_{p_0}^2}{c^2}\left[\frac{n}{\gamma}\right]\mathbf{E} - \frac{\omega_{p_0}^2}{c^2}\mathbf{E} + \nabla(\nabla\cdot\mathbf{E})\ . \tag{9}$$

In Eq. (9) the group velocity V_g is taken as $V_g = c$, the term $\nabla(\nabla\cdot\mathbf{E})$ is of order δ^2 smaller than the others and may be neglected. The third term on the rhs appears from the cancellation in the left-hand side of the linear contribution of the dielectric constant $\varepsilon_e = 1 - (\omega_{p_0}^2/\omega^2) = (k^2c^2/\omega^2)$, for an assumed preformed plasma prior to the pulse arrival. For $\delta \ll 1$, γ is given by the QSA and so does n_2 on the τ scale. We thus take the value of n_2, given by Eq. (8), into Eq. (9) to describe the temporal evolution of the pulse.

The experimental situation where intense lasers are focused in a plasma requires a realistic description of the central part of the beam where the main contribution to self-focusing occurs. This region is well described by the paraxial approximation, where the field is written as $\mathbf{E} = E(r,z,t)e^{iS(r,z,t)}\hat{\epsilon}$, with $S \equiv [(1/f)df/dz]r^2 + \varphi(z)$, where f represents the change of the beam radius where $\hat{\epsilon}$ is the polarization direction. For the calculations, we assume an initial Gaussian (space-time) pulse

$$E^2(r,z,\eta) = E_{00}^2(\eta)\exp\left[-\frac{r^2}{r_0^2f^2(\eta,z)}\right]\ , \tag{10}$$

$$E_{00}^2(\eta) = E_{00}^2(0)\exp\left[-\frac{\eta^2}{\tau^2}\right]\ , \tag{11}$$

with $\eta = t - z/V_g$. All quantities of γ, E, η are expanded to order $r^2/r_0^2f^2 \ll 1$, with r_0 being the initial beam radius.

We also consider a parabolic model for the electronic density,

$$n_0(r) = n_0(0)\left[1 - \sigma\frac{r^2}{L_n^2}\right]\ , \tag{12}$$

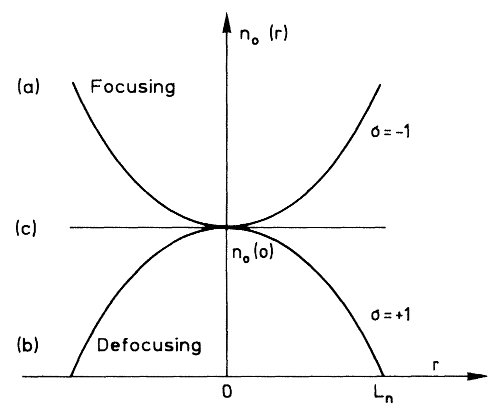

FIG. 1. Parabolic radial profile of the electronic density, $n_0(r) = n_0(0)\ [1 - \sigma(r^2/L_n^2)]$ for (a) a concave profile ($\sigma = -1$) leading to focusing, and (b) a convex profile ($\sigma = +1$) leading to defocusing, and (c) a homogeneous profile.

with $\sigma=+1$ (-1) for a plasma acting as a defocusing (focusing) lens (Fig. 1). The differential equations satisfied by the focusing radius (f) and the phase (φ) are

$$\frac{1}{f}\frac{d^2f}{dz^2}=\frac{\alpha_e^2}{f^4}+\frac{\sigma\delta^2B^2}{\gamma}-\frac{\delta^2E_0^2(\eta)}{2\gamma^3f^4}-\frac{4\alpha_e^2E_0^2(\eta)}{\gamma^2}\left[\frac{1}{f^6}+\frac{\sigma B^2}{f^4}\right]+\frac{4\alpha_e^2E_0^4(\eta)}{\gamma^4f^8}, \tag{13}$$

$$\frac{d\varphi}{dz}=-\frac{\alpha_e^2}{f^2}+\frac{\delta^2}{2}\left[1-\frac{1}{\gamma}\right]+\frac{\alpha_e^2E_0^2(\eta)}{2\gamma^2f^4}, \tag{14}$$

where $L_e=r_0$; $B\equiv L_e/L_n$; $E_0^2(\eta)\equiv E_{00}^2(\eta)/2E_c^2$, and z is given in units of r_0.

In Eq. (13) the first term on the rhs is related to the vacuum diffraction, and the second is related to the zero-order electronic density inhomogeneity. This term may be either focusing $(\sigma=-1)$ or defocusing $(\sigma=+1)$. The third term is the focusing effect due to the relativistic mass increase. The fourth has two contributions of the ponderomotive mechanism, one related to the term $\nabla\cdot\mathbf{f}_p\simeq\nabla^2\gamma$ and the other from the coupling of the ponderomotive force with ∇n_0 (which can be focusing or defocusing, depending on σ). The last term comes from the contributions of $\nabla^2\gamma$ and $|\nabla\gamma|^2/\gamma$, plus a further contribution from the mass increase which is the same as that due to $|\nabla\gamma|^2/\gamma$.

From Eq. (13), we obtain a very simple expression for the critical power, valid to order $(\alpha/\delta)^4$, at the entrance of the plasma $(f=1)$,

$$\mathcal{P}^*=\mathcal{P}_H^*\left[1+\sigma\left[\frac{\delta B}{\alpha}\right]^2\right]=\mathcal{P}_H^*\left[1+\sigma\frac{n(0)}{\Delta n}B^2\right], \tag{15}$$

where $\Delta n=1/4\pi r_e r_0^2$, $r_e=q_e^2/m_0c^2$ is the classical electron radius, and $\mathcal{P}_H^*$ is the critical power for the homogeneous plasma. $\mathcal{P}_H^*\simeq10^{10}(\omega/\omega_{p_0})^2W$. The experimental laser and plasma parameters (favorable to self-focusing) lead to values in the range $\alpha=0.01-0.03$ and $\delta=0.1-0.25$. Equation (15) shows that if the plasma behaves as a defocusing lens $(\sigma=+1)$, a variation of a few percent in the degree of the plasma homogeneity may greatly increase the value of the critical power. This is easily verified since $\Delta n=2.8\times10^{17}$ electrons/cm^3, for $r_0=10$ μm and $\lambda=1$ μm, and $n(0)\gtrsim10^{19}$ electrons/cm^3. This strong defocusing effect has been experimentally observed by Auguste *et al.* [9].

Figure 2(a) shows the behavior of the critical power as a function of the plasma inhomogeneity. The straight dashed line represents the initial beam power $\mathcal{P}_L$ chosen to be below $\mathcal{P}_H^*$. The defocusing (focusing) situation, $\mathcal{P}_L/\mathcal{P}^*<1$ $(\mathcal{P}_L/\mathcal{P}^*>1)$, is represented by points I and II (III, IV, and V) which correspond to Figs. 2(b) I and 2(b) II [Figs. 2(b) III, 2(b) IV, and 2(b) V], respectively.

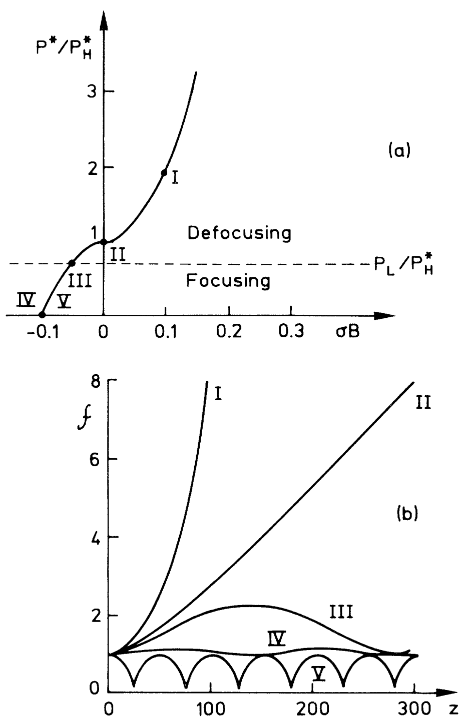

FIG. 2. (a) Behavior of the critical power $(\mathcal{P}^*/\mathcal{P}_H^*)$ as a function of the inhomogeneity (σB); $\alpha/\delta=0.1$. The chosen initial laser power $\mathcal{P}_L=0.75\mathcal{P}_H^*$ (straight dashed line). (b) I, II, III, IV, V. The beam radius (f) as a function of the axial coordinate z (in units of r_0) for $\sigma B=0.1$; 0; -0.05; -0.1; -0.1, respectively. $\mathcal{P}_L=0.75\mathcal{P}_H^*$ except for 2(b) V where $\mathcal{P}_L=7.5\mathcal{P}_H^*$. Curves III and IV show that in spite of a laser power $\mathcal{P}_L$ less than the critical power $\mathcal{P}_H^*$ for a homogeneous plasma, self-focusing begins to take place when the radial profile of the electron density has a minimum on the laser axis as shown in Fig. 1(a). Moreover, curve V, which corresponds to another condition where the laser power is larger than $\mathcal{P}_H^*$, gives rise to a stronger focusing behavior.

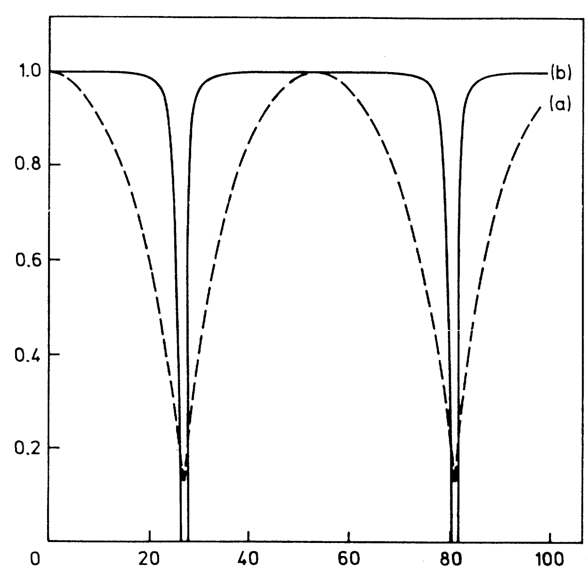

FIG. 3. The beam radius (a) and the electronic density (b) as a function of the axial coordinate (z in units of r_0). $E_0^2=0.1$; $\alpha=0.03$; $\delta^2=0.05$; $B=0$.

The striking effect of tailoring the plasma into a convergent lens ($\sigma=-1$) is shown in Figs. 2(b) III–2(b) V which correspond, respectively, to the onset of the beam capture, to the quasisoliton propagation, and to the guided propagation at $\mathcal{P}_L=7.5\mathcal{P}_H^*$. The condition for guided propagation $B^2=\Delta n/n_0 \Longrightarrow \mathcal{P}^*=0$ is similar to that obtained by Sprangle and Esarey [10].

Figure 3 shows the electronic density and the variation of the beam radius as a function of z. There is a significant depletion of the electronic density only at the minimum values of f. If $f^2 \ll 1$, we may have $\alpha_e^2 E_0^2 f^4 \gg 1$ and the perturbative expansion on α_e is no longer valid. To account for strong density depletion (cavitation) one has to compute higher-order term in α_e^{2n} for δn_e.

Figure 4 clearly shows that the capture of the laser beam depends on its angle of incidence on the plasma. The beam which is focused in Fig. 4(b) is defocused in 4(a). This same behavior is observed for $E^2 \simeq 10E_c^2$ for which we will have $\alpha_e^2 E_0^2/f^4 < 1$, which validates the perturbative approach at these field values.

In conclusion our main results are the following:

(i) Plasma inhomogeneity plays a major role in self-focusing. Analytical results show that a variation of a few percent of the degree of homogeneity of the plasma in its central region may greatly modify the value of the critical power with respect to the homogeneous case. For the present, the preformed plasmas are produced by laser atomic ionization and behave as a defocusing lens. We suggest a tailoring of the preformed plasma into a focusing lens in order to achieve the conditions favorable to the experimental observation of the effect. This can be obtained by electronic densities which are minimized at the beam axis, leading also to optical guiding of a Gaussian pulse in a parabolic density profile.

(ii) The capture of the light beam depends on its angle of incidence in the plasma.

(iii) The diffractional erosion of the leading and trailing edge of the pulse is observed.

(iv) The mechanisms responsible for self-focusing are the electron-mass increase and the plasma inhomogeneity. The ponderomotive force plays a secondary role in this process.

(v) Some previous models [5–7] may be regarded as special cases of this perturbative scheme. Even though a first attempt has been recently made [11] new experimental studies of relativistic self-focusing are urgently required in order to confirm the promising theoretical calculations.

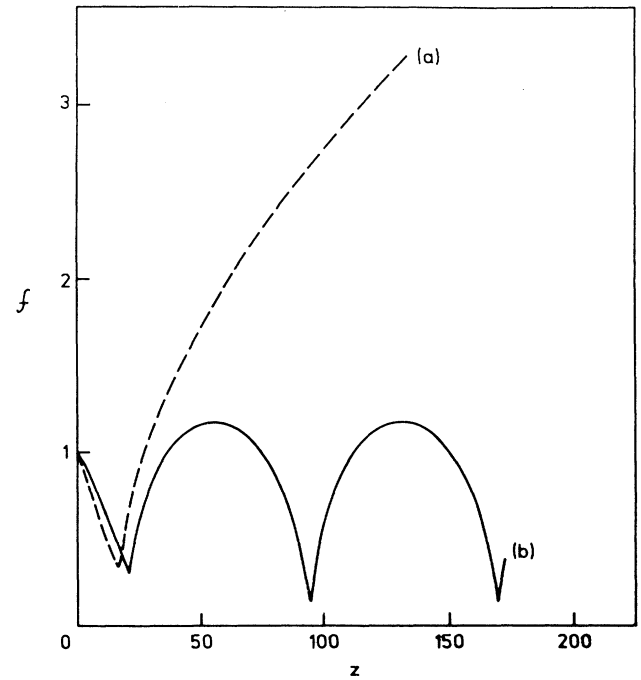

FIG. 4. The beam radius as a function of the axial coordinate (z in units of r_0). $E_0^2=0.1$; $\alpha=0.03$; $\delta^2=0.05$; $B=0$. (a) $df/dz=-0.04$ represents the strong convergent angle of incidence at the entrance in the plasma, and (b) $df/dz=-0.02$ represents the weak convergent angle of incidence.

Very recently two important publications concerning short pulse propagation in relativistic plasmas have appeared. The first is a numerical two-dimensional determination of propagation and guiding of intense pulses including wake-field effects [12], and the second deals with a study of the dynamical coupling of density n_2 with the electric-field envelope **E** in the weak relativistic regime [13].

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[1] P. Maine *et al.*, IEEE J. Quantum Electron. **34**, 398 (1988).
[2] M. Ferray *et al.*, Opt. Commun. **75**, 278 (1990).
[3] C. Sauteret *et al.*, Opt. Lett. **16**, 238 (1991).
[4] T. Auguste *et al.*, in *The Tenth International Conference on Laser Spectroscopy,* edited by M. Ducloy, E. Giacobino, and G. Camy (World Scientific, Singapore, 1991), p. 341.
[5] G. Z. Sun, E. Ott, Y. C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).
[6] D. C. Barnes, T. Kurki-Suonio, and T. Tajima, IEEE Trans. Plasma Sci. **PS-15**, 154 (1987); T. Kurki-Suonio, P. J. Morrison, and T. Tajima, Phys. Rev. A **40**, 3230 (1989).
[7] A. B. Borisov *et al.*, Phys. Rev. A **45**, 5830 (1992); A. B. Borisov *et al.*, Phys. Rev. Lett. **65**, 1793 (1990); Zh. Eksp. Teor. Fiz. **101**, 1132 (1992) [Sov. Phys. JETP **74**, 604 (1992)].
[8] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Zh. Eksp. Teor. Fiz. **50**, 1537 (1966) [Sov. Phys. JETP. **23**, 1025 (1966)].
[9] T. Auguste *et al.*, Opt. Commun. **89**, 145 (1992).
[10] P. Sprangle and E. Esarey, Phys. Fluids B **4**, 2241 (1992).
[11] A. B. Borisov *et al.*, Phys. Rev. Lett. **68**, 2309 (1992).
[12] P. Sprangle *et al.*, Phys. Rev. Lett. **69**, 2200 (1992).
[13] T. M. Antonsen and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).